Interest Calculations

The change in the value of money over time as a result of its earning power is called the time value of money. The earning power of money can be considered as a cost of using money. Interest can be defined as money paid for the use of borrowed money.

Money cannot be shifted through time without changing its value. Generally, when money is moved forward in time, its value increases through compounding. When it is moved backward, its value decreases through discounting. Two separate sums of money can only be compared if they are evaluated at the same point in time. Otherwise, they must be converted to an equivalent time base.

The TK Solver Library includes a collection of functions useful for calculations involving interest. The factors that affect the equivalence are the amount of the cash flow, the timing of the cash flow, and the interest rate.

Simple Interest

Simple interest is calculated by multiplying the principal amount of money by the interest rate per period and the number of periods involved. The interest charge is based only on the principal sum which does not include any accumulated interest charges. Simple interest can be expressed as

I = P x i x N

I is the simple interest charge; P is the present sum of money; i is the interest rate per period; N is the number of periods.

Example: An engineer borrows $20,000 at a simple interest rate of 4% per year for three years. Compute the interest charge that must be paid by the engineer at the end of the three years.

I = P x i x N
= 20000 x 0.04 x 3
= 2400

Compound Interest

Compound interest for any given period is calculated based on the remaining original principal amount plus any accumulated interest charges up to the beginning of that period.

Example: The engineer in the previous example chooses not to pay any interest charge until the termination of the loan at the end of the third year and ends up paying interest on the interest retained. Compute the total amount that must be paid at the end of the third year if the interest is compounded annually.

At the beginning of year 1, $20,000 is owed. $20,000 x 0.04 = $800 is the interest charge for year 1, so the amount owed at the end of year 1 is $20,800. During the second year, the interest charge is

$20,800 x 0.04 = $832. The total amount owed at the end of year 2 is $20,800 + $832 = $21,632. In the third and final year, the interest charge is $21,632 x 0.04 = $865.28 bringing the total amount owed to $21,632 + $865.28 = $22,497.28.

Discrete Compound Interest Formulas

Compounding is the mathematical process of periodically adding return to principal and thereby increasing the principal on which future return is based. In other words, the compounding process is a mathematical tool by which the future equivalent value of a present sum can be found at a particular interest rate. The future equivalent is called the future worth or future value of the present sum.

Similarly, discounting is the mathematical process to reduce a principal given at some future time to its equivalent at the present time. The discounting process is a mathematical tool by which the present equivalent of a future sum can be found at a specific interest (discount) rate. Discounting is the inverse of the compounding process. The present equivalent found by this process is defined as the present worth or value of a future sum.

Nomenclature:

r = nominal interest rate per period
i = effective interest rate per period
n = number of compounding periods
P = present value
F = future value
A = end-of-period cash flows, or equivalent end-of-period values, in a uniform series continuing for a specified number of periods. The letter A implies “annual” or “annuity”.
G = Uniform period-by-period increase or decrease in cash flows or amounts. The letter G represents an arithmetic gradient.

Unless specifically stated otherwise, it is customary to assume that all cash flows are flowing discretely at the end of the given period.

Single-Payment Compound Amount Factor (Finding F, Given P)

If an amount P is deposited now in an account earning an effective interest rate of i% per period compounded per period, then the account at the end of period n can be expressed as

$$F = P \cdot (1 + i)^n$$

Since each of the variables in this equation appears just once, TK Solver can solve the equation for any scenario in which all but one variable has a known value. Given P=20000, i=0.04, n=3, TK computes F=22497.28. Given P=20000, F=24000, n=3, TK computes i=0.06266.

Single-Payment Present Value Factor (Finding P, Given F)
This factor is simply the inverse of the previous factor.

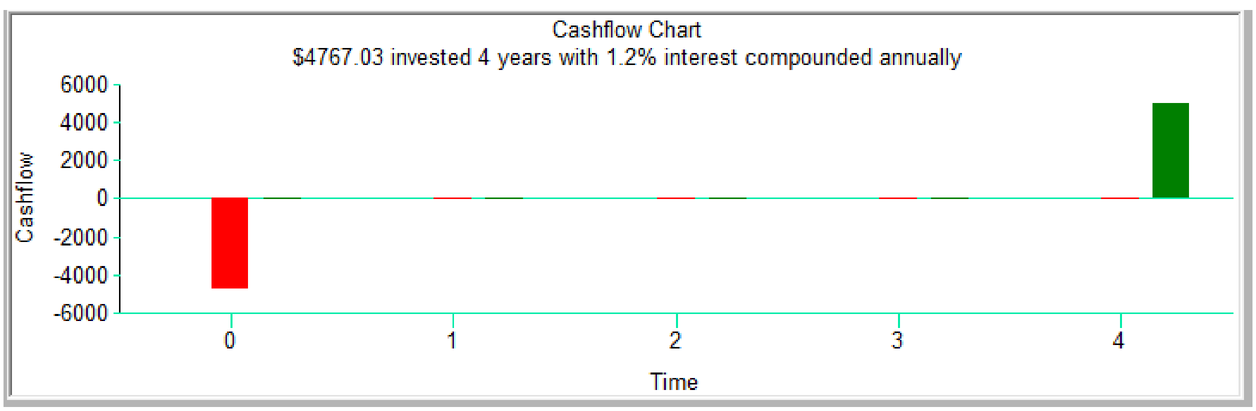
$$P = F \cdot \left[\frac{1}{(1+i)^n} \right]$$

Example 1: Determine the present value that when deposited into a savings account for 4 years at an interest rate of 1.2% compounded annually will have a future value of $5,000.

Solution: The equation P = F/(1+i)^n can be used. Solving with inputs F=5000, i=0.012, n=4 produces P=4767.03.

Here is the cashflow table and chart for this example.

Time	Cash Out	Cash In
0	-4767.03	0
1	0	0
2	0	0
3	0	0
4	0	5000

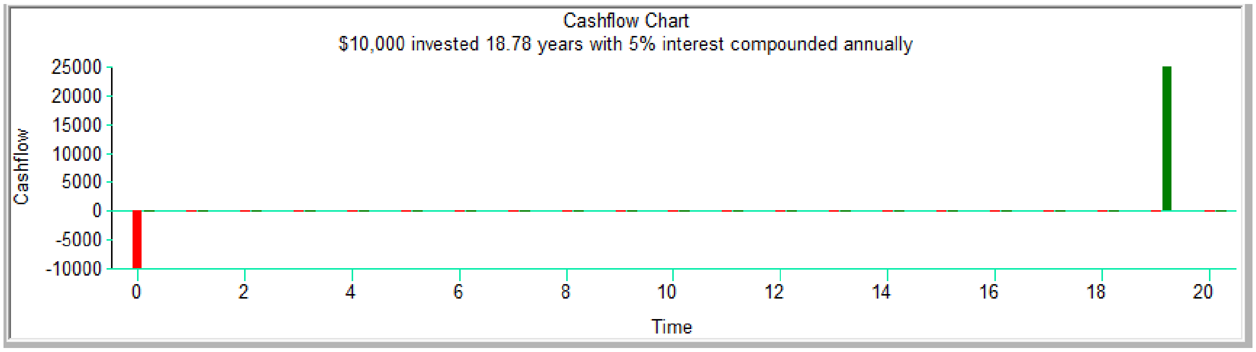


Example 2: Determine the number of years that a $10,000 investment, earning 5% annually, must be kept in order to accumulate $25,000.

Solution: The equation F = P*(1+i)^n can be used. Solving with inputs F=25000, P=10000, i=0.05, produces n=18.78.

Here is the cashflow table and chart for this example.

Time	Cash Out	Cash In
0	-10000	0
1	0	0
2	0	0
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	0	0
9	0	0
10	0	0
11	0	0
12	0	0
13	0	0
14	0	0
15	0	0
16	0	0
17	0	0
18	0	0
18.78	0	25000



Uniform-Series Compound Amount Factor (Finding F, Given A)

A series of uniform (equal and equally spaced amounts) cash flows of A occurring at the end of each period for n periods with interest rate i% per period is often called an annuity. The future value of an annuity can be found using the following equation.

$$F = A \cdot \left[\frac{(1+i)^{n} - 1}{i} \right]$$

Notice that the variable i appears twice in this equation. This means that the equation must be solved iteratively if i is the output. Fortunately, TK Solver is well-equipped for that. All other variables can be solved for directly.

Uniform-Series Sinking Fund Factor (Finding A, Given F)

The inverse of the prior factor can be used to find A, given F.

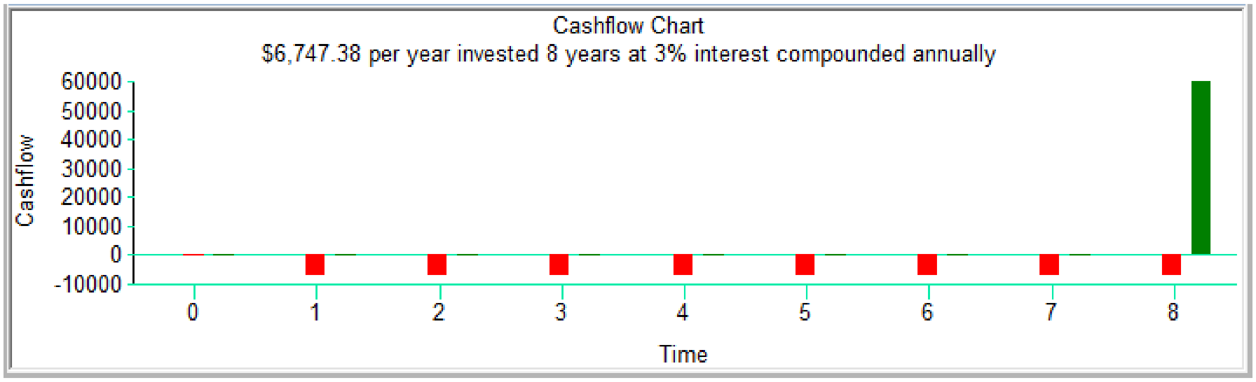
$$A = F \cdot \left[\frac{i}{(1+i)^{n} - 1} \right]$$

Example 3: A company just purchased production equipment for $60,000 with a useful life of 8 years. Management desires to establish a fund to put aside a certain amount from the net income produced by the equipment at the end of each year for the next 8 years in order to replace the equipment at the end of its useful life. This is often called a sinking fund. If the fund earns 3% interest how much should the company deposit at the end of each year?

The rule A = F*(i/((1+i)^n-1)) solves the problem. Given F=60000, i=0.03, n=8, TK computes A=6747.38.

Here is the cashflow table and chart summarizing this example.

Time	Cash Out	Cash In
0	0	0
1	-6747.38	0
2	-6747.38	0
3	-6747.38	0
4	-6747.38	0
5	-6747.38	0
6	-6747.38	0
7	-6747.38	0
8	-6747.38	60000



Uniform-Series Present Worth Factor (Finding P, Given A)
Given a uniform-series cash flow of amount A occurring at the end of each period for n periods when the effective interest rate is i%, we determine the present worth in the following way.

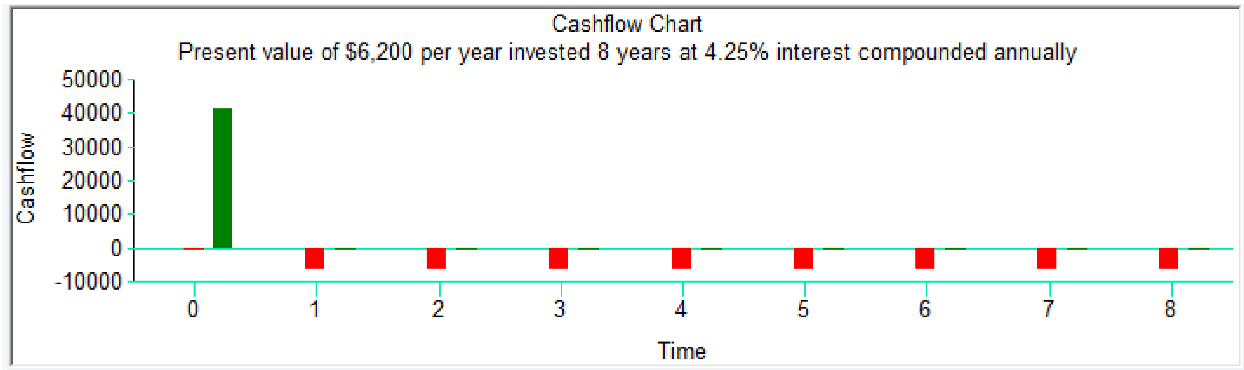
$$P = A \cdot \left[\frac{(1+i)^n - 1}{i \cdot (1+i)^n} \right]$$

Example 4: Find the present value of the sinking-fund amount of $6,200 for the next 8 years, assuming an effective interest rate of 4.25%.

Solution: The rule P = A*((((1+i)^n-1))/(i*(1+i)^n)) will solve it, given the inputs A=6200, i=0.0425, n=8. P=41315.45.

Here is the cashflow table and chart summarizing this example.

Time	Cash Out	Cash In
0	0	41315.45
1	-6200	0
2	-6200	0
3	-6200	0
4	-6200	0
5	-6200	0
6	-6200	0
7	-6200	0
8	-6200	0

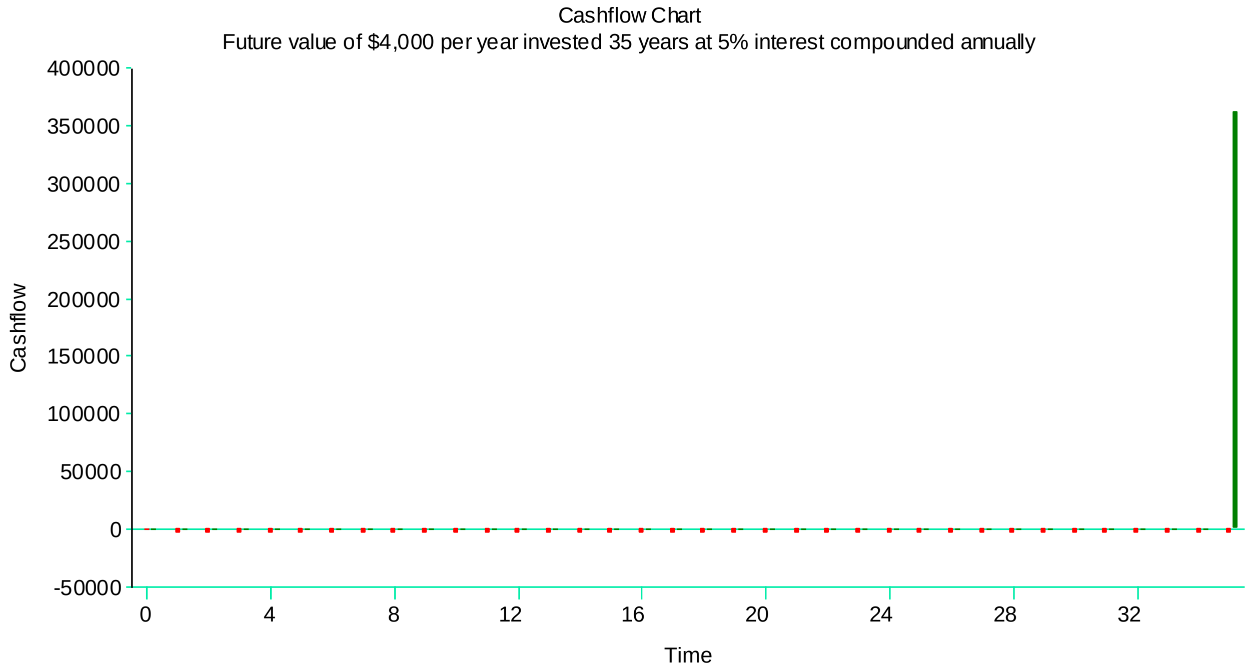


Example 5: A 30-year-old engineer wants to establish a tax-sheltered annuity plan for her retirement. Her yearly end-of-year premium is $4,000 earning 5% interest compounded annually. Determine the total savings that will be accumulated after 35 years.

Solution: We use the equation F = A*(((1+i)^n-1)/i) with inputs A=4000, i=0.05, n=35 and solve for F=361281.23. That is, the 35*4000 = 140000 in savings grows to 361281.23 through 5% compounding over 35 years.

Here is a portion of the cashflow table and chart summarizing this example.

Time	Cash Out	Cash In
24	-4000	0
25	-4000	0
26	-4000	0
27	-4000	0
28	-4000	0
29	-4000	0
30	-4000	0
31	-4000	0
32	-4000	0
33	-4000	0
34	-4000	0
35	-4000	361281.23



Example 6: Using the data from Example 5, assume that the engineer has just made her 35th payment today. Determine the present value of her total savings accumulated so far. Note that 15 years ago (just after her 20th payment) the interest rate was increased to 7.5%.

Solution: As a first step, the worth of the total savings after the 20th payment can be found from the equation F = A*(((1+i)^n-1)/i) with inputs A=4000, i=0.05, n=20. F = 132263.82.

Therefore, the total savings accumulated, including today's payment, is found from the equation

F = P20*(1+i2)^n2 + A*(((1+i2)^n2-1)/i2) .

Since P20=132263.82, i2=0.075, n2=1, A=4000, F = 495825.88 .

Uniform-Series Capital Recovery Factor (Finding A, Given P)
This is also known as the amortization factor. It is the inverse of the prior factor.

$$A = P \cdot \left[\frac{i \cdot (1+i)^n}{(1+i)^n - 1} \right]$$

Example 7: Assume that a 30-year old engineer plans to use $683,410 savings in a tax-sheltered annuity plan to provide uniform annual payments as an annuity for 20 years after her retirement at age 65. Determine the amount of the annuity that she will be receiving at the end of each year if the interest rate is assumed to be 4.25% during her retirement years.

Solution: The equation A = P*((i*(1+i)^n)/(((1+i)^n-1))) with inputs P=683410, i=0.0425, n=20 returns A = 51405.99.

Here is the cashflow table and chart for this example.

Time	Cash Out	Cash In
0	0	683410
1	-51405.99	0
2	-51405.99	0
3	-51405.99	0
4	-51405.99	0
5	-51405.99	0
6	-51405.99	0
7	-51405.99	0
8	-51405.99	0
9	-51405.99	0
10	-51405.99	0
11	-51405.99	0
12	-51405.99	0
13	-51405.99	0
14	-51405.99	0
15	-51405.99	0
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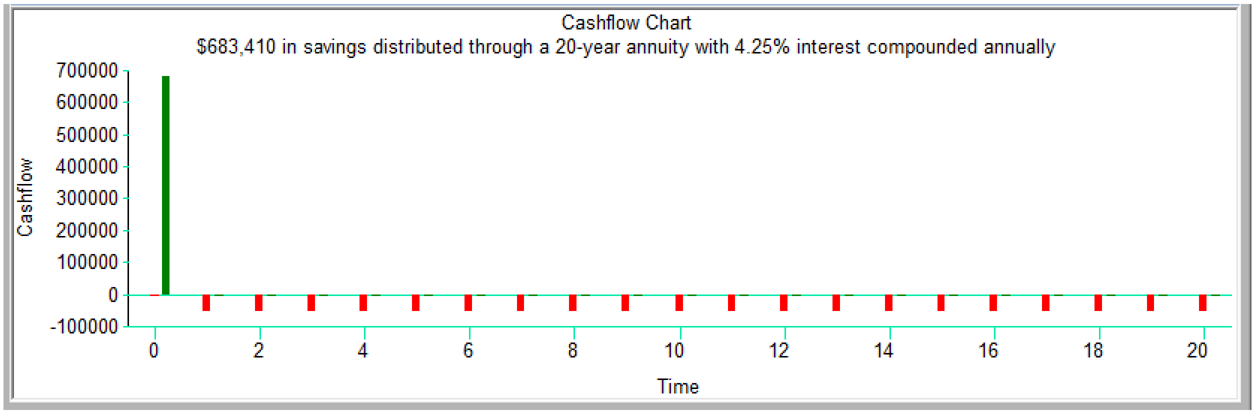
17	-51405.99	0
18	-51405.99	0
19	-51405.99	0
20	-51405.99	0



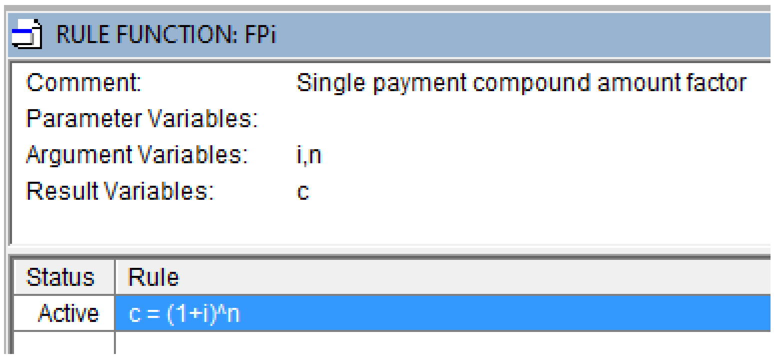
Example 8: Based on the previous example, assume that the projected inflation rate is 2.6%. Compute the present worth of her first annuity payment.

Solution: The annuity was found to be $51,405.99, which is a future value with respect to the present time. It will happen in 36 years. We use the equation P1 = A*(1/(1+inflation)^n1), with inputs A=51405.99, inflation=0.026, and n1=36. We get P1 = 20403.69.

It is important to note that this is actually the single-payment present worth equation even though we are using variable A. A was the previously computed annuity value. This potential confusion can be overcome by implementing TK Solver rule functions to represent each of the factors. Rule functions use local variables with the core equations and those local variables are mapped to the variables in the problems at hand. This also simplifies future problem-solving tasks since rule functions can be referenced multiple times in a single equation.

TK Solver Rule Functions for Compound Interest Factors
We introduce the following TK Solver rule functions which also serve as a review of the formulas covered to this point. The function names are based on the required output and the assumed input and type of interest applied. Each function includes a descriptive comment along with the local argument and result variable names used in the function's rule.

Function **FPi** has argument variables i and n, and result variable c.

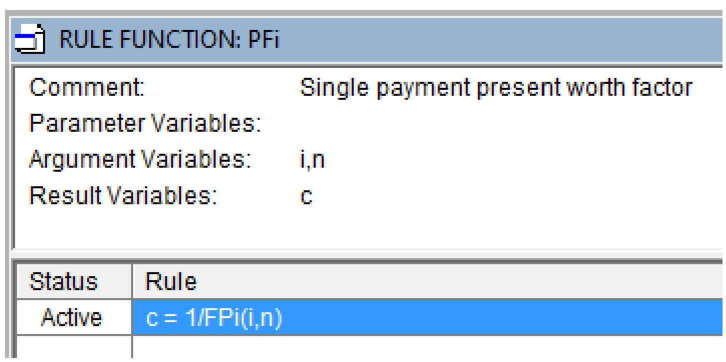


Function FPi can be applied in solving for present or future values. For example, the equation

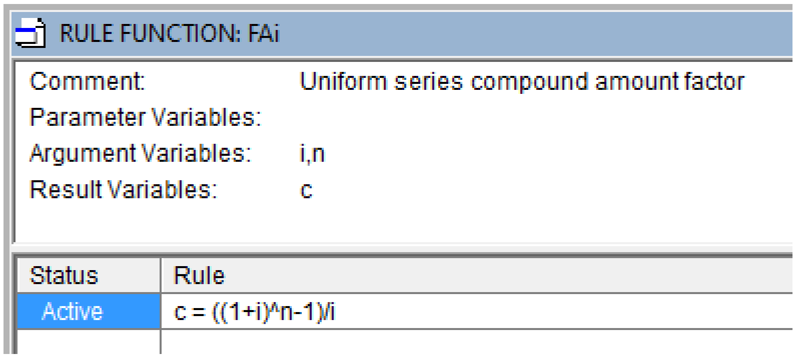
F = P*FPi(i,n)

can be solved for any of the four variables.

Although FPi is invertible for each of the variables, a redundant function representing the inverse is useful. Such a function can be referenced from procedure functions where statements are processed with respect to specific inputs. It is simple to create function **PFi** for this purpose.

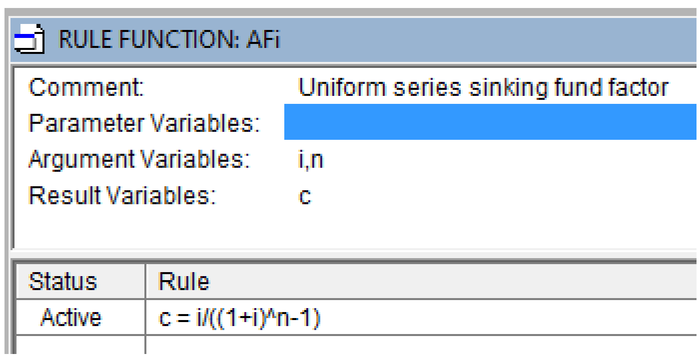


We add rule function **FAi** to our collection.

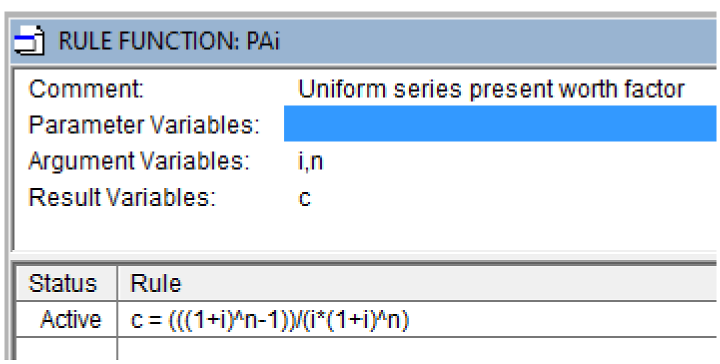


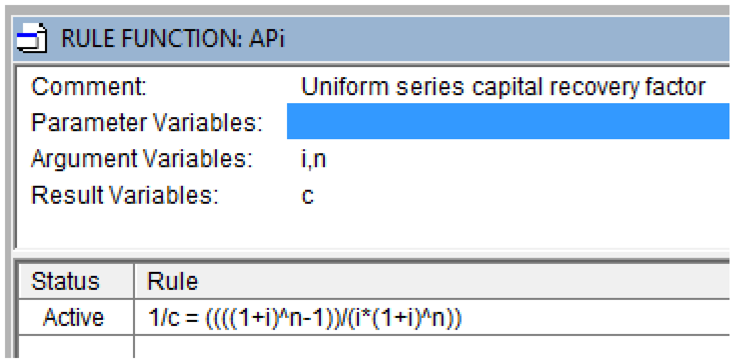
The equation F = A*Fai(i,n) solves for the future value of an annuity.

Rule function **AFi** computes the uniform series sinking fund factor.



We add rule functions **PAi** and **APi** the collection.



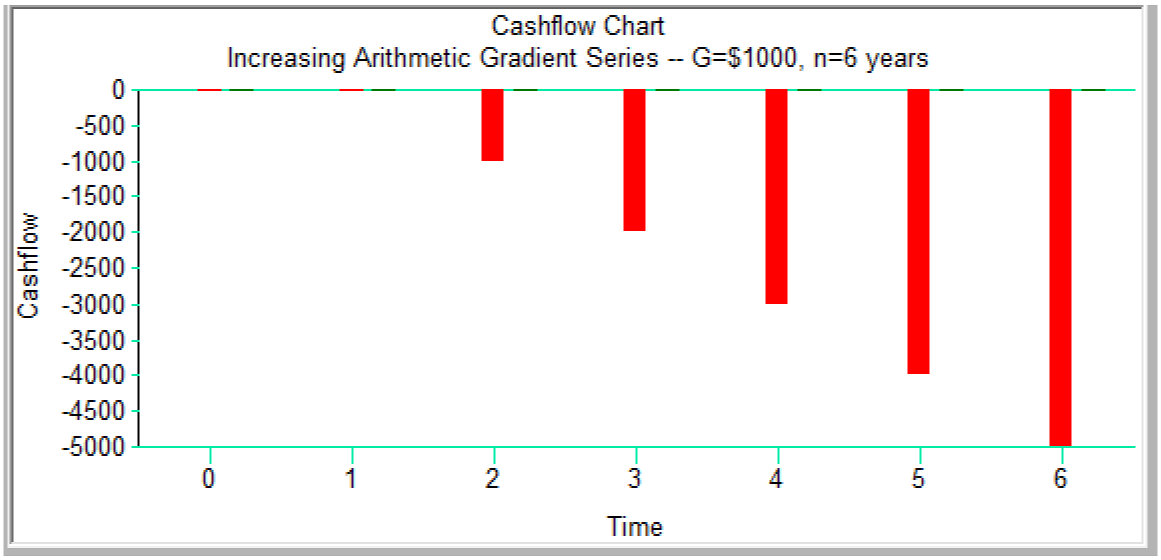


Example 9: We return to Examples 7 and 8 now to see how our rule functions are used to solve those problems. Recall that there was potential confusion regarding the variable names. A 30-year old engineer plans to use $683,410 savings in a tax-sheltered annuity plan to provide uniform annual payments as an annuity for 20 years after her retirement at age 65. Determine the amount of the annuity that she will be receiving at the end of each year if the interest rate is assumed to be 4.25% during her retirement years. Assume that the projected inflation rate is 2.6%. Compute the present worth of her first annuity payment.

Solution: The equation, A = P*APi(i,n) with inputs P=683410, i=0.0425, n=20 returns A = 51405.99. This solves the amount of the annuity payments. Next, we use P1 = A*PFi(inflation,n1) with inputs inflation=0.026 and n1=36. The output value of A from the first equation is applied in this equation. The resulting P1 value is 20403.69.

Arithmetic Gradient Series

Some situations my involve disbursements or receipts that are projected to increase or decrease by a constant amount from period to period. Examples include equipment maintenance and repair expenses and depreciation expense calculations. The following cashflow chart shows a series of end-period disbursements increasing at a constant amount of change, G, per year.



The equation used to compute the future value of such a series of cash flows with an assumed interest rate per period i is shown here.

$$F = G \cdot \left[\frac{1}{i} \cdot \left[\frac{(1+i)^n - 1}{i} - n \right] \right]$$

The present value of such a series of cash flows can then be computed using the PFi function. Alternatively, the present value can be computed directly using the following equation.

$$P = G \cdot \left[\frac{1}{i} \cdot \left[\frac{(1+i)^n - 1}{(1+i)^n \cdot i} - \frac{n}{(1+i)^n} \right] \right]$$

We created TK Solver rule functions for each of these along with their inverses using similar nomenclature as used for the uniform-series functions.

Example 10: Assume that certain end-of-year expenses are $0 at the end of the first year, $1,200 for the second year, $2,400 for the third year and $3,600 for the fourth year. If the interest rate is 5% per year, determine the future value of the expenses that will be spent by the end of the fourth year.

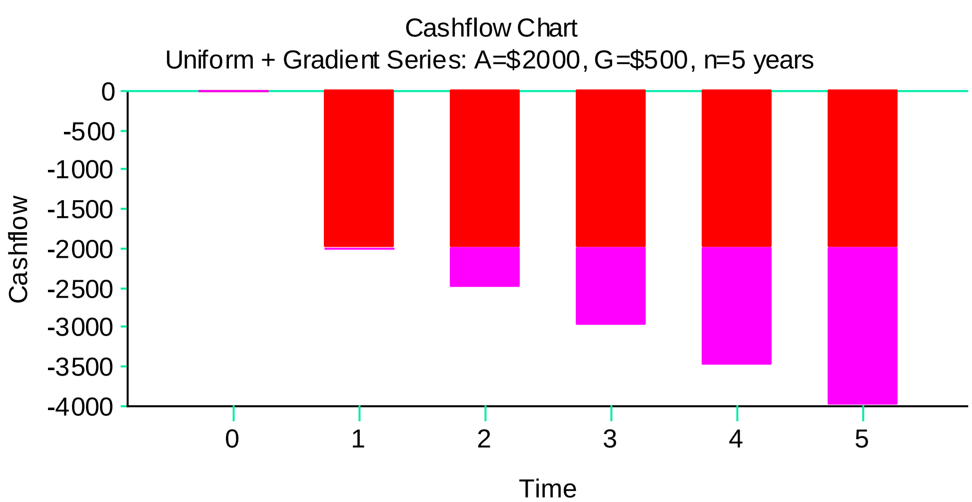
Solution: The equation F = G*FGi(i,n) is used with inputs G=1200, i=0.05, n=4 produces F=7443.

Example 11: The operating and maintenance costs of a piece of equipment are expected to be $2,000 for the first year, $2,500 the second year, $3,000 the third year, $3,500 the fourth year and $4,000 the fifth year. Assume an interest rate of 5.5% per year and the costs at the end of the given year to determine the equivalent future value of the arithmetic gradient series.

Solution: This can be solved by noting that we have a uniform series of $2,000 cash flows and a gradient series with $500 increments. We can then sum the results of the two. We have F = F1 + F2 where F1=2000*FAi(0.055,5) and F2=500*FGi(.055,5). These produce F1=11162.18, F2=5282.65 and F=16444.83.

Here is a cashflow table and chart for this example.

Time	Uniform	Gradient
0	0	0
1	-2000	0
2	-2000	-500
3	-2000	-1000
4	-2000	-1500
5	-2000	-2000



Example 12: Determine the equivalent uniform annual worth of the cashflows in Example 11.

Solution: We use the equation A = F*AFi(i,n) with inputs F=16444.83, i=0.055, n=5 to produce A=2946.53.

Geometric Gradient Series

When disbursements or receipts are projected to increase or decrease by a constant percentage (exponentially) from one period to the next, they are described as a geometric gradient series. Examples include some maintenance costs, labor costs, material costs, and energy costs. The geometric gradient series can also be used to determine the effects of a constant rate of inflation or deflation.

Three different formulas are required to determine the present value of geometric series, based on the relationship between the constant percentage change (g) and the interest rate (i). A special interest rate (i`) must be introduced and the present value is a function of g, i, i`, n, and the initial value of the series (A1). The first case is when g is greater than i.

$$\text{if } g > i \text{ then } i` = \frac{1+g}{1+i} - 1$$

$$\text{if } g > i \text{ then } P = \frac{A1}{1+i} \cdot \left[\frac{(1+i`)^n - 1}{i`} \right]$$

The second case is when g is less than i.

$$\text{if } g < i \text{ then } i` = \frac{1+i}{1+g} - 1$$

$$\text{if } g < i \text{ then } P = \frac{A1}{1+g} \cdot \left[\frac{(1+i`)^n - 1}{i` \cdot (1+i`)^n} \right]$$

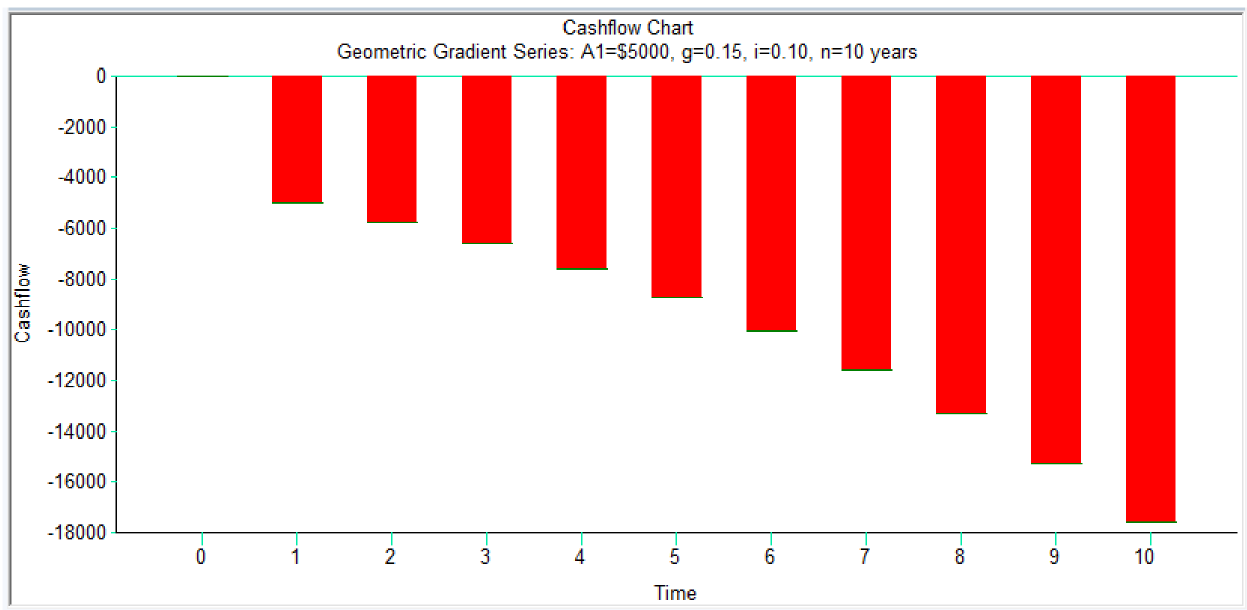
The third case is when g = i.

$$\text{if } g = i \text{ then } P = \frac{A1 \cdot n}{1+g}$$

Example 13: The current annual maintenance cost of equipment is given as $5,000 and it is expected to increase at the rate of 15% per year over the next ten years. Assuming an interest rate of 10%, compute the present worth of the maintenance costs.

Here is a table and chart of the cash flows.

Time	Outflows
0	0
1	-5000
2	-5750
3	-6612.5
4	-7604.37
5	-8745.03
6	-10056.79
7	-11565.3
8	-13300.1
9	-15295.11
10	-17589.38



Solution: Since g>i, we use the case 1 formulas with inputs g=0.15, i=0.10, n=10, A1=-5000 and get the results i`=0.04545… and P=-55973.76.

Continuous Interest Compounding Factors

Interest rates are normally stated on an annual basis. For example, an interest rate may be quoted as 4% compounded quarterly. Here the 4%, understood to be an annual rate, is called the nominal interest rate. Since a year is divided into four quarters, the interest rate per period is 1% per quarter. Therefore, the nominal interest rate is the annual interest rate disregarding the effects of compounding where the periods are less than one year. The nominal annual interest rate, r, can be expressed as r = i x M where i is the effective interest rate per period and M is the number of compounding periods per year.

The effective interest rate is the annual rate including the effects of compounding at the end of periods shorter than one year. Therefore, the effective interest rate is

$$i_{eff} = (1 + i)^{M} - 1$$

or alternatively,

$$i_{eff} = \left[1 + \frac{r}{M}\right]^{M} - 1$$

If compounding is continuous, M is infinity and we have

$$i_{eff} = e^{(r)} - 1$$

The effective interest rate can also be expressed as a function of present and future values.

$$i_{eff} = \left[\frac{F}{P}\right]^{\left[\frac{1}{n}\right]} - 1$$

Comparison of Nominal and Effective Rates
Here is a table illustrating how the effective rate increases as M (compounding periods per year) increases, using a nominal rate of 18%.

M	Effective %
1	18
2	18.81
4	19.2518
12	19.5618
52	19.6845
365	19.7164
infinity	19.7217

Example 14: Assume that an engineer borrows $10,000 for a three-year period. The loan will be repaid at the end of the three years with a single payment of $16,430. What is the effective interest rate?

Solution: We know the present and future values and the term, so we use

$$i_{eff} = \left[\frac{16430}{10000}\right]^{\left[\frac{1}{3}\right]} - 1$$

and get 0.18 or 18% for the effective interest rate.

Example 15:
A college student wants to buy a used car. The total price of the car is $6,000, including tax. A down payment of $1,000 is required. The car dealer recommends borrowing the remaining amount of $5,000 from the credit agency next door. They offer a 3-year loan to be paid in 36 monthly installments. The interest on the loan is 1.75% per month. The credit agency also charges $40 for a credit investigation fee and because of the student's age and limited credit history, requires the student to purchase private risk insurance for $900. Determine the monthly payments and the effective interest rate on the total loan.

Solution: The total loan amount is $5,000 + $40 + $900 = $5,940. Therefore, the total interest will be 5940*36*0.0175 = 3742.20. So the total amount to be repaid is $5,940 + $3,742.20 =$ 9,682.20. Dividing by 36, we get the monthly installment amount, $268.95. We use the iPA function in TK Solver to get the monthly interest rate, i = iPA(5000/268.95,36) = 0.041228379639533. Then we compute the annual effective interest rate as 1.041228379639533^12 – 1 = 0.6239 or 62.39%. That's a very expensive loan!

Continuous Compounding Interest Formulas

When interest is compounded continuously, the effective interest rate is equal $e^r - 1$, where r is the nominal interest rate. Substituting this expression into the discrete compound interest factors gives us the corresponding continuous compounding interest formulas. For example, the formula relating future value and present value was earlier given as

$$F = P \cdot (1 + i)^n$$

Substituting, this becomes

$$F = P \cdot e^{(r \cdot n)}$$

Since all four of the variables appear just once in that equation, TK Solver can solve for any of them if the other three are known. As before with the discrete period compounding functions, we added the individual rule functions to the compound interest collection to allow them to be referenced directly from procedure functions as necessary.

Uniform Series Continuous Compounding

Substitution in the uniform series sinking fund factor produces the equation

$$A = F \cdot \left[\frac{e^{(r)} - 1}{e^{(r \cdot n)} - 1} \right]$$

This is called the discrete uniform-series continuous compounding sinking-fund factor. Similarly, we have

$$F = A \cdot \left[\frac{e^{(r \cdot n)} - 1}{e^{(r)} - 1} \right]$$

This is called the discrete uniform-series continuous compounding compound amount factor.

The following TK Library functions are added to the collection for processing continuous compounding factors with single or discrete series of cash flows. In addition to the core functions, we have included inverse functions to solve for interest rates and time durations.

FPr	Rule	2;1	Single payment continuous compound amount factor
nFPr	Rule	2;1	Inverse of FPr function, returning n
rFP	Rule	2;1	Inverse of FPr function, returning r
PFr	Rule	2;1	Single payment continuous present worth factor
nPFr	Rule	2;1	Inverse of PFr function, returning n
rPF	Rule	2;1	Inverse of PFr function, returning r
FAr	Rule	2;1	Discrete uniform-series continuous compounding compound amount factor
nFAr	Rule	2;1	Inverse of FAr function, returning n
rFA	Procedure	2;1	Inverse of FAr function, returning r
AFr	Rule	2;1	Discrete uniform-series continuous compounding sinking fund factor
nAFr	Rule	2;1	Inverse of AFr function, returning n
rAF	Procedure	2;1	Inverse of AFr function, returning r
PAr	Rule	2;1	Continuous compounding present worth factor
rPA	Procedure	2;1	Inverse of PAr function, returning r
nPAr	Procedure	2;1	Inverse of PAr function, returning n
APr	Rule	2;1	Continuous compounding capital recovery factor
rAP	Procedure	2;1	Inverse of APr function, returning r
nAPr	Procedure	2;1	Inverse of APr function, returning n

Example 16:
Assume that you will be making a uniform series of year-end deposits of $5,000 each for the next 15 years at a 5% interest rate compounded continuously. Determine the effective interest rate and the present value of the uniform payments.

Solution: The effective interest rate is $i = e^{0.05} - 1 = 0.051271$. The present worth of the uniform payments is P = 5000*PAr(0.05,15) = 51455.25.

Example 17: Using the details from the previous example, (a) determine the future value of the deposits at the end of the 15th year. (b) Then assume that the first deposit is made now instead of at the end of the year and the remaining deposits are made at the end of every year for the next 14 years. Determine the future value of the deposits at the end of the 15th year.

Solutions: (a) The future value is found from F = 5000*FAr(0.05,15) = 108930.77. (b) We find the future value in 14 years and then find the future value of that value one year later. The solution is found from the equation F = (5000*FPr(0.05,14) + 5000*FAr(0.05,14))*FPr(.05,1) = 114515.77.

Continuous Compounding and Continuous Cash Flows
In some economic scenarios the assumption of continuous cash flows instead of discrete ones would be more realistic. A new set of compound interest factors is required.

We introduce a new variable, A`, representing the money flowing continuously and uniformly during each period. We then have the following formulas for future values and present values of such cash flows.

$$F = A` \cdot \left[\frac{e^{(r \cdot n)} - 1}{r} \right]$$

$$P = A` \cdot \left[\frac{e^{(r \cdot n)} - 1}{r \cdot e^{(r \cdot n)}} \right]$$

As with all prior interest factors, we also include TK Solver Library functions representing the inverse functions for computing A`, r, or n when inputs for the rest of the parameters are given. Here is the complete collection of functions for continuous compounding and continuous cash flows.

FA`r	Rule	2;1	Continuous uniform-flow continuous compounding compound amount factor
nFA`r	Rule	2;1	Inverse of FA`r function, returning n
rFA`	Procedure	2;1	Inverse of FA`r function, returning r
A`Fr	Rule	2;1	Continuous uniform-flow continuous compounding sinking-fund factor
nA`Fr	Rule	2;1	Inverse of A`Fr function, returning n
rA`F	Procedure	2;1	Inverse of A`Fr function, returning r
PA`r	Rule	2;1	Continuous uniform-flow continuous compounding present value factor
nPA`r	Rule	2;1	Inverse of PA`r function, returning n
rPA`	Procedure	2;1	Inverse of PA`r function, returning r
A`Pr	Rule	2;1	Continuous uniform-flow continuous compounding capital recovery factor
nA`Pr	Rule	2;1	Inverse of A`Pr function, returning n
rA`P	Procedure	2;1	Inverse of A`Pr function, returning r
PF`r	Rule	2;1	Single cont. uniform-payment cont. compounding present worth factor
nPF`r	Rule	2;1	Inverse of PF`r function, returning n
rPF`	Procedure	2;1	Inverse of PF`r function, returning r
F`Pr	Rule	2;1	Single cont. uniform-payment cont. compounding compound amount factor
nF`Pr	Rule	2;1	Inverse of F`Pr function, returning n
rF`P	Procedure	2;1	Inverse of F`Pr function, returning r

Example 18:
Assume that there will be a uniform series of continuous expenditures totaling $5,000 annually for the next 7 years and that the interest rate will be compounded continuously at 12% per year and determine the equivalent present value and future value of the series.

Solution: The equivalent present value is P = 5000*PA`r(.12,7) = 23678.73.

The future value is 5000*PA`r(.12,7) = 54848.62.