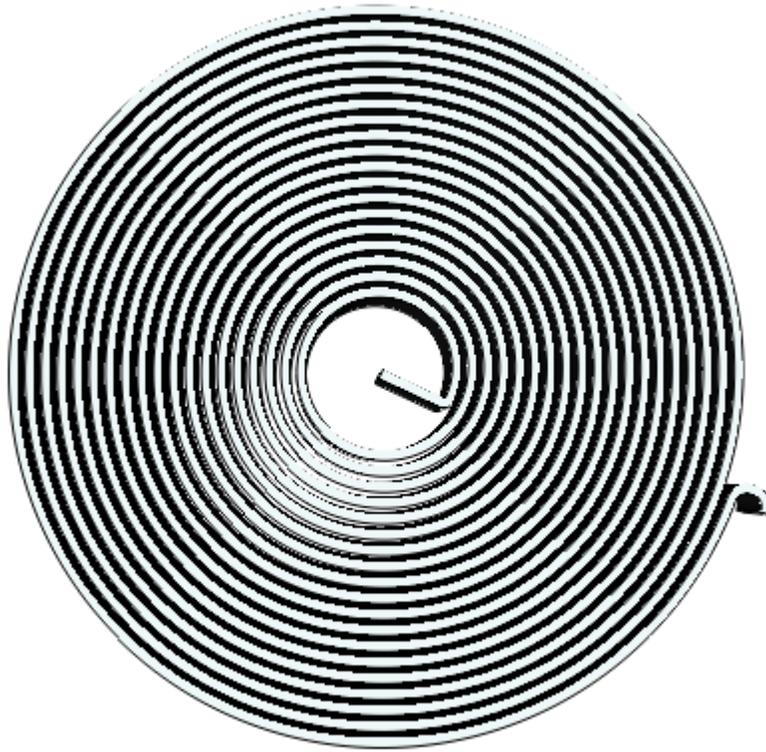


Case Study: Iterative Solutions of Spiral Torsion Spring Equations

The nonlinear equations involved in designing spiral torsion springs can produce some surprising results. The simultaneous nonlinear equations must be solved iteratively for some scenarios. Our Advanced Spring Design (ASD7) software has the equation solving power of TK Solver. This article gives some insights on what TK is doing behind the scenes. Here is what a spiral torsion spring looks like.



Assume the following constraints:

High-strength 1050 carbon steel strip will be used, having a bending modulus (BMod) of $3E7$.

Spring width (Dw) = 0.500"

Arbor diameter (ID) = 0.700"

Minimum free diameter (OD) = 3.0"

Torque to solid (Pmax) = 15 lbf-in

Design torque (P) = 12 lbf-in

Design deflection (Defl) = 3 turns

In this scenario, the strip thickness (dt) and active length (Lact) are unknown. The design equations can be solved iteratively using an initial guess for the strip thickness. Here is the first equation which solves for the active strip length (Lact).

$$\text{Defl} = \frac{6 \cdot P \cdot \text{Lact}}{\pi \cdot D_w \cdot dt^3 \cdot B_{\text{Mod}}}$$

Next the free coils (FC) are computed.

$$\text{Lact} = \frac{\text{FC} \cdot \pi \cdot (\text{OD} + \text{ID})}{2}$$

Next is the strip area.

$$\text{StripArea} = \text{Lact} \cdot dt$$

Next is the solid diameter.

$$\text{SolidDia} = \sqrt{\frac{4 \cdot \text{StripArea}}{\pi} + \text{ID}^2}$$

Next is the number of solid coils.

$$\text{SolidCoils} = \frac{\text{SolidDia} - \text{ID}}{2 \cdot dt}$$

Next is the available turns (Nt).

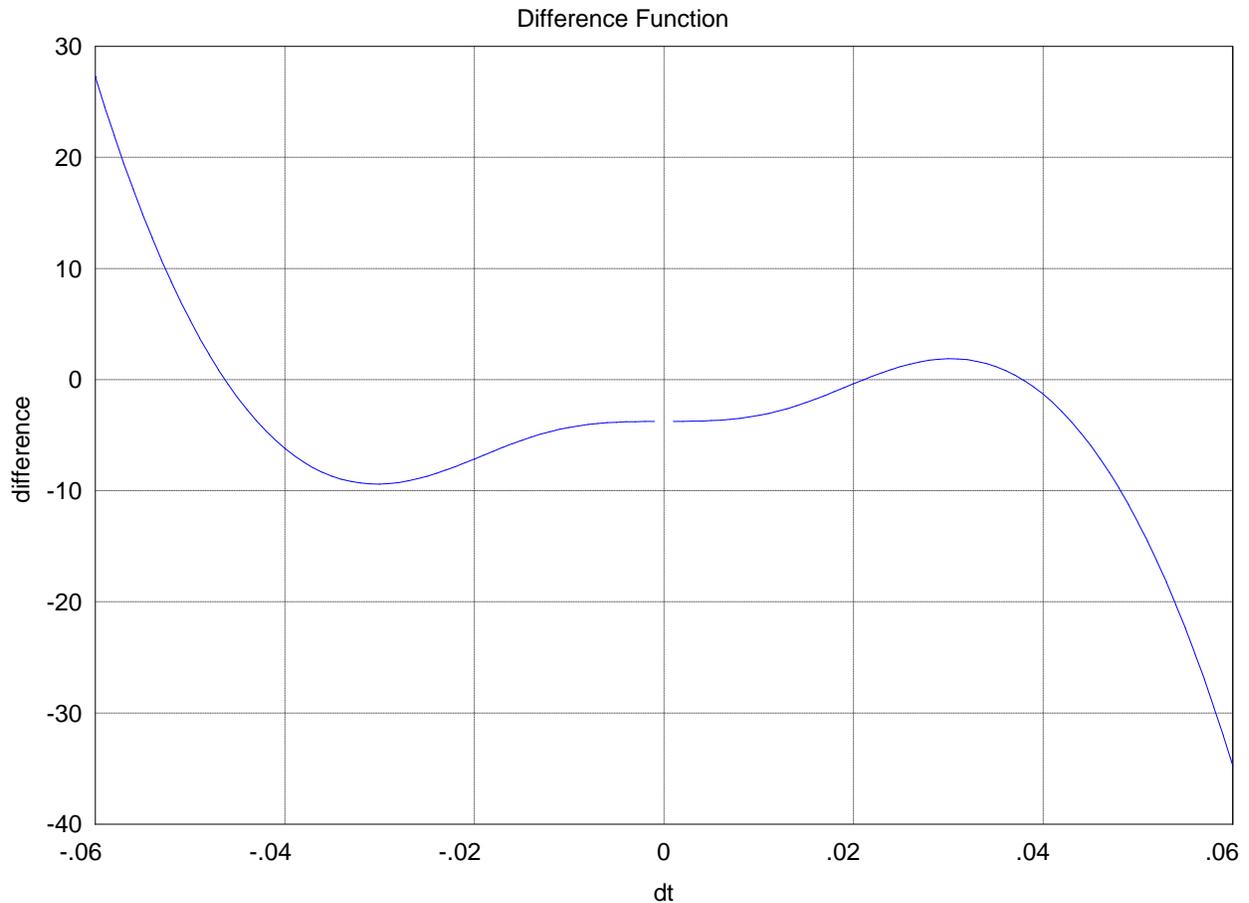
$$\text{Nt} = \text{SolidCoils} - \text{FC}$$

At that point, the next equation becomes overdefined, with all the variables having known or computed values but with the left hand side not equal to the right.

$$\text{Nt} = \frac{6 \cdot P_{\text{max}} \cdot \text{Lact}}{\pi \cdot D_w \cdot dt^3 \cdot B_{\text{Mod}}}$$

TK Solver keeps trying values for the strip thickness (dt) until the sequence of equations results in a difference between the left and right sides of the last equation that is essentially zero.

Here is a plot of the difference as a function of dt. This plot reveals that there are three values of dt that result in the difference being 0. One is around -0.047. Another is near 0.021 and another is near 0.038. That difference function is quite a numerical roller coaster!



The choice of the starting point in searching for the solution is very important. The iterative process uses the slope of this difference function at the point of the initial guess to determine its next guess. Suppose we guess $dt = 0.015$. The iterative solution is 0.0210610 . Suppose we guess $dt = 0.050$. The iterative solution is 0.0378558 . Suppose we guess $dt = 0.027$. Which way will it go? It finds the solution to the left. Suppose we guess $dt = 0.032$. Which way will it go? It finds the solution to the right.

These solutions result in two very different spiral torsion spring designs. Suppose $dt = 0.0210610$ ". From the first equation, we see that $Lact=18.3429$ ". On the other hand, if $dt = 0.0378558$ " then $Lact=106.5194$ ". That strip is almost six times longer!

Here is the first solution in ASD7, with the strip thickness input and torque to solid output.

Torsion Spring - Spiral

Material: Med. Carbon 1050

Strip Strength: Moderate High Extra High

Input / Output Arrangements: Power User Quick Start

Note: *Italicized labels indicate optional inputs.* Assume Uniform Coil Spacing

Arbor Diameter (Di)	0.7000	in	Design Limits:		
Min Free Diameter (Do)	3.0000	in	Torque to Set	5.803	lbf-in
Spring Width (b)	0.5000	in	Turns to Set	1.4509	
Spring Thickness (t)	0.0210610	in	Torque to solid	15.000	lbf-in
Thickness Tolerance	0.00075	in	Turns to Solid	3.7500	
Width/Thickness Ratio	23.74		Solid Coils	7	
Active Length	18.3429	in	Free Coils	3.1561	
Length/Thickness Ratio	871		Load Condition:		
% of Fill Area	5.8		Torsional Moment	12.000	lbf-in
Spring Weight	0.0551	lb	Deflection	3.000	turns
Minimum Tensile Strength (MTS)	224293	psi	Stress	324642	psi
Spring Rate	4.0000	lbf-in/rev	Corrected Stress at OD	321893	psi
Estimated Cycle Life	<1E4		Corrected Stress at ID	327438	psi

Design Status

Warning(s):

1. Deflection exceeds turns to set.
2. Stress exceeds Tensile Strength.
3. Life estimate is for reference only. Consult your spring maker for guidance.
4. b/t ratio > 15
5. Torque to Solid > Torque to Set

DXF Drawing for Current Spring Design

DXF Text Height: 0.14 Refresh All dimensions are in inches

Here is the second solution in ASD7, with the strip thickness input and torque to solid output.

Torsion Spring - Spiral

Material: Med. Carbon 1050

Strip Strength: Moderate High Extra High

Input / Output Arrangements: Power User Quick Start

Note: *Italicized labels indicate optional inputs.* Assume Uniform Coil Spacing

Arbor Diameter (Di)	0.7000	in	Design Limits:		
Min Free Diameter (Do)	3.0000	in	Torque to Set	17.814	lbf-in
Spring Width (b)	0.5000	in	Turns to Set	4.4535	
Spring Thickness (t)	0.0378558	in	Torque to solid	15.000	lbf-in
Thickness Tolerance	0.00100	in	Turns to Solid	3.7500	
Width/Thickness Ratio	13.21		Solid Coils	22	
Active Length	106.5190	in	Free Coils	18.3276	
Length/Thickness Ratio	2814		Load Condition:		
% of Fill Area	60.3		Torsional Moment	12.000	lbf-in
Spring Weight	0.5746	lb	Deflection	3.000	turns
Minimum Tensile Strength (MTS)	213096	psi	Stress	100484	psi
Spring Rate	4.0000	lbf-in/rev	Corrected Stress at OD	98965	psi
Estimated Cycle Life	>1E7		Corrected Stress at ID	102050	psi

Design Status

Warning(s):

1. Life estimate is for reference only. Consult your spring maker for guidance.

DXF Drawing for Current Spring Design

DXF Text Height: 0.14 Refresh All dimensions are in inches

In both solutions, the torque to solid output is the desired value, 15. Everything else is very different.

The math model used in ASD7 returns the solution with the thicker strip. This is accomplished by starting with a relatively large guess for that dimension. This latter solution will tend to provide a spring design that is less likely to be overstressed in the fully wound condition.