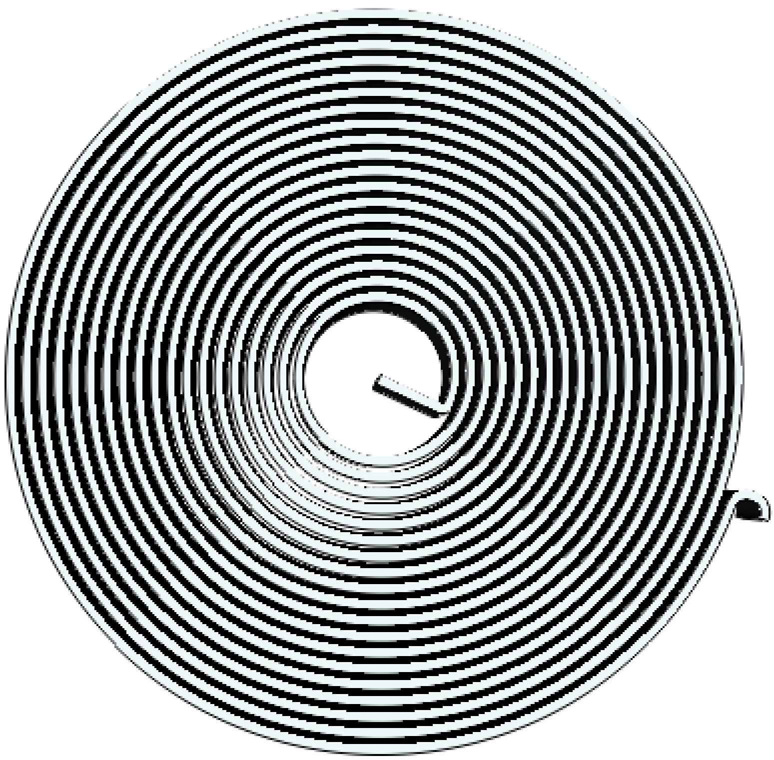
**Case Study: Iterative Solutions of Spiral Torsion Spring Equations**

The nonlinear equations involved in designing spiral torsion springs can produce some surprising results. The simultaneous nonlinear equations must be solved iteratively for some scenarios. Our Advanced Spring Design (ASD7) software has the equation solving power of TK Solver. This article gives some insights on what TK is doing behind the scenes. Here is what a spiral torsion spring looks like.



Assume the following constraints:

High-strength 1050 carbon steel strip will be used, having a bending modulus (BMod) of 3E7.
Spring width (Dw) = 0.500”
Arbor diameter (ID) = 0.700”
Minimum free diameter (OD) = 3.0”
Torque to solid (Pmax) = 15 lbf-in
Design torque (P) = 12 lbf-in
Design deflection (Defl) = 3 turns

In this scenario, the strip thickness (dt) and active length (Lact) are unknown. The design equations can be solved iteratively using an initial guess for the strip thickness. Here is the first equation which solves for the active strip length (Lact).

$$Defl = \frac{6 \cdot P \cdot Lact}{\pi \cdot Dw \cdot dt^3 \cdot BMod}$$

Next the free coils (FC) are computed.

$$Lact = \frac{FC \cdot \pi \cdot (OD + ID)}{2}$$

Next is the strip area.

StripArea = Lact * dt

Next is the solid diameter.

$$SolidDia = \sqrt{\frac{4 \cdot StripArea}{\pi} + ID^2}$$

Next is the number of solid coils.

$$SolidCoils = \frac{SolidDia - ID}{2 \cdot dt}$$

Next is the available turns (Nt).

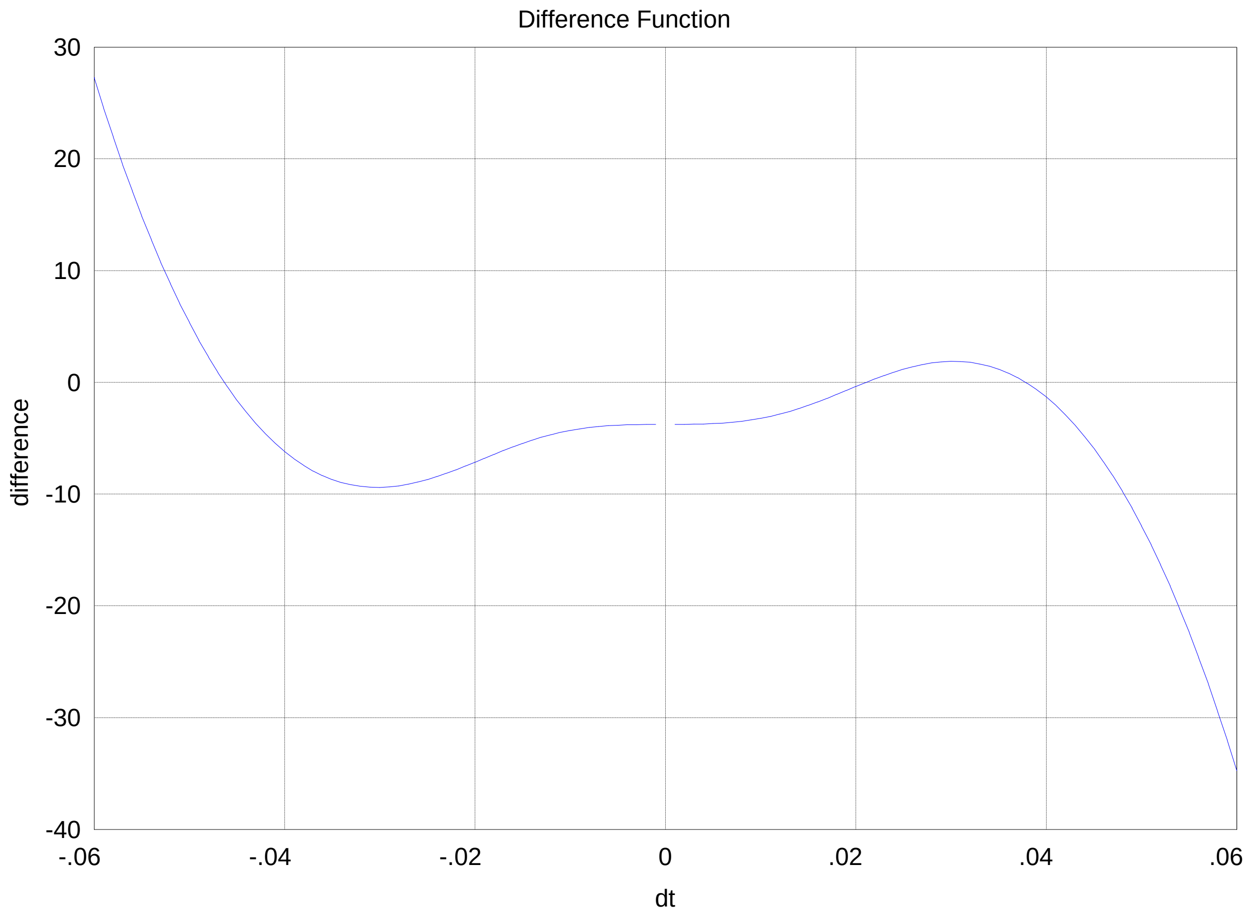
Nt = SolidCoils - FC

At that point, the next equation becomes overdefined, with all the variables having known or computed values but with the left hand side not equal to the right.

$$Nt = \frac{6 \cdot Pmax \cdot Lact}{\pi \cdot Dw \cdot dt^3 \cdot BMod}$$

TK Solver keeps trying values for the strip thickness (dt) until the sequence of equations results in a difference between the left and right sides of the last equation that is essentially zero.

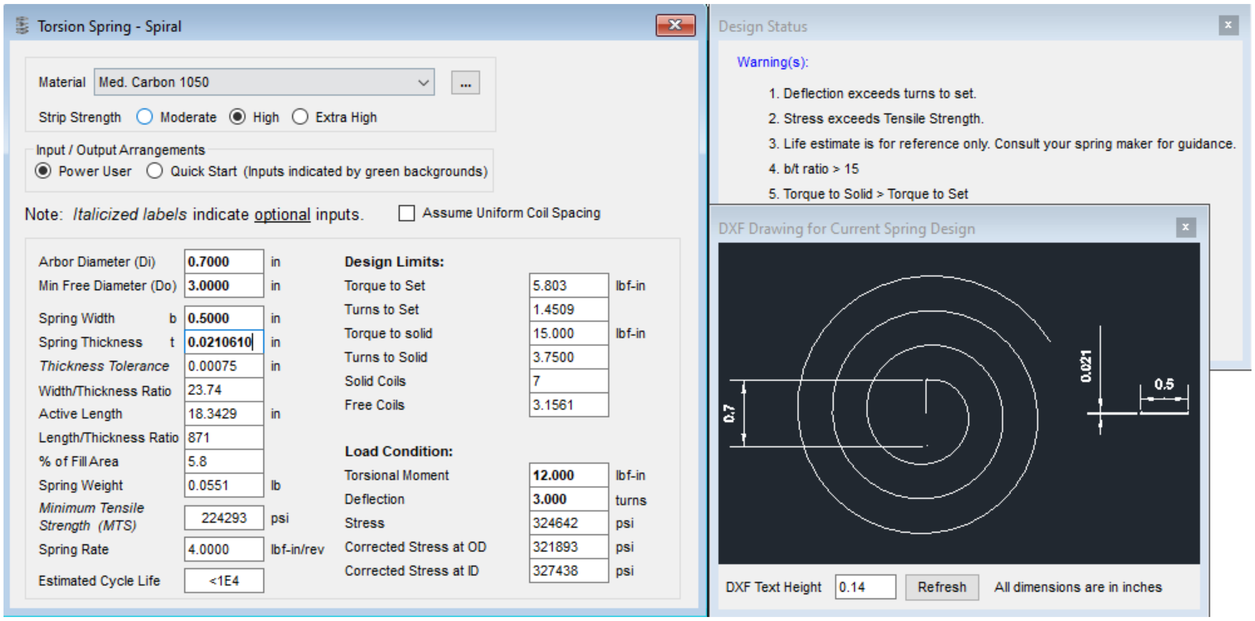
Here is a plot of the difference as a function of dt. This plot reveals that there are three values of dt that result in the difference being 0. One is around -0.047. Another is near 0.021 and another is near 0.038. That difference function is quite a numerical roller coaster!



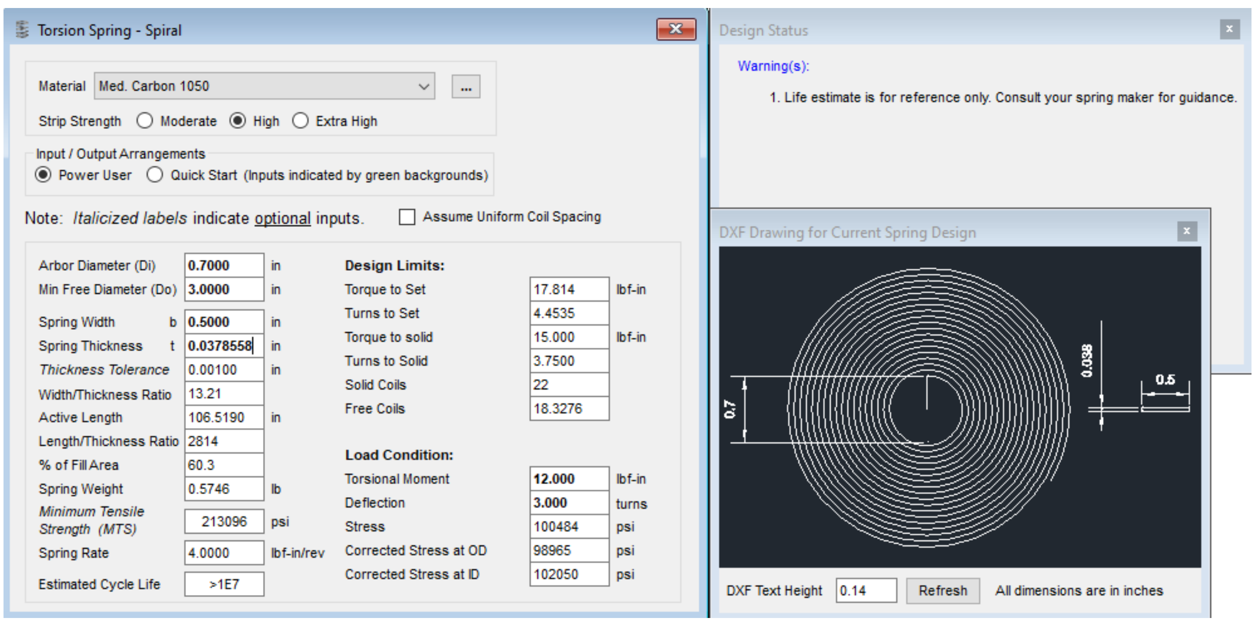

The choice of the starting point in searching for the solution is very important. The iterative process uses the slope of this difference function at the point of the initial guess to determine its next guess. Suppose we guess dt = 0.015. The iterative solution is 0.0210610. Suppose we guess dt = 0.050. The iterative solution is 0.0378558. Suppose we guess dt = 0.027. Which way will it go? It finds the solution to the left. Suppose we guess dt = 0.032. Which way will it go? It finds the solution to the right.

These solutions result in two very different spiral torsion spring designs. Suppose dt = 0.0210610”. From the first equation, we see that Lact=18.3429”. On the other hand, if dt = 0.0378558” then Lact=106.5194”. That strip is almost six times longer!

Here is the first solution in ASD7, with the strip thickness input and torque to solid output.




Here is the second solution in ASD7, with the strip thickness input and torque to solid output.




In both solutions, the torque to solid output is the desired value, 15. Everything else is very different.

The math model used in ASD7 returns the solution with the thicker strip. This is accomplished by starting with a relatively large guess for that dimension. This latter solution will tend to provide a spring design that is less likely to be overstressed in the fully wound condition.