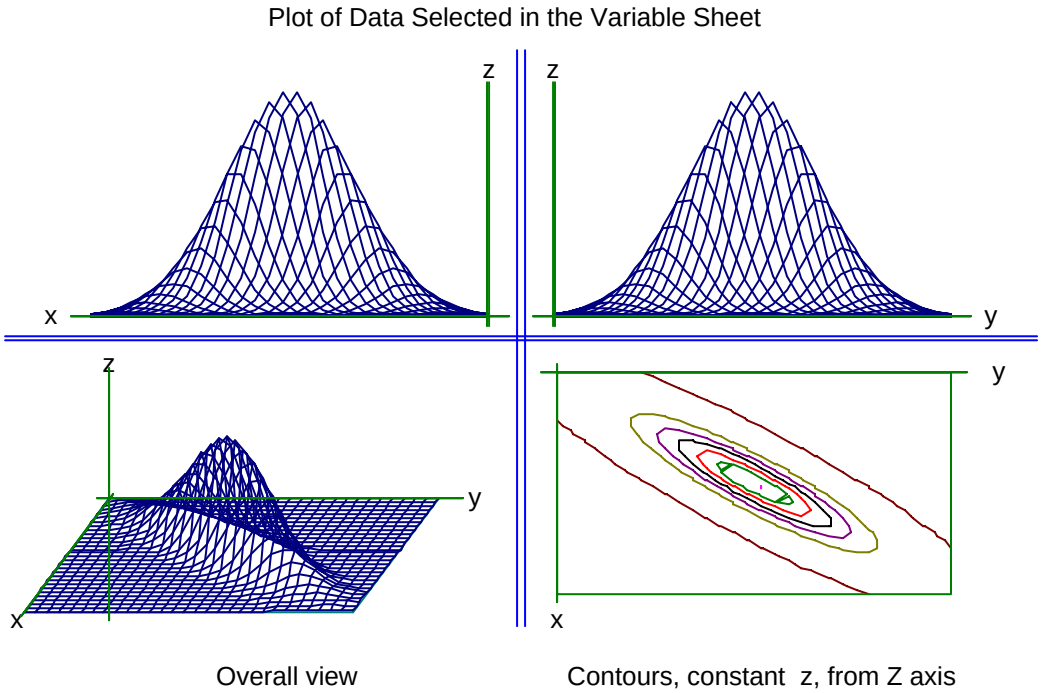
Case Study: Bivariate Normal Probabilities in Engineering

An extensive study was made of shear strength (x2) and weld diameter (x1) in an attempt to substitute a less expensive, nondestructive testing procedure for a destructive test. The results of the study were as follows:

For the weld diameter, the mean and variance were 0.20 in and 0.02 in^2.
For the shear strength, the mean and variance were 1100 lbf and 525 lbf^2.
The correlation between x1 and x2 was 0.90.
The joint distribution was assumed to be approximately bivariate normal.

The TK Solver Library includes a model for doing probability calculations involving bivariate normal distributions. Load the model from the Statistics section, under Distributions and Random Numbers and input the values given above. The inputs for the standard deviations (s1 and s2) are the square roots of the variances.

Here is a plot of the overall distribution in four views.



Based on this information, we can ask a number of potentially interesting questions.

Question 1: What is the probability of observing a weld diameter less than 0.5 inches and a shear strength of less than 1150 pounds?

St	Input	Name	Output	Unit	Comment
	0.2	m1			Mean of x1
	0.14142136	s1			Standard deviation of x1
	1100	m2			Mean of x2
	22.9128785	s2			Standard deviation of x2
	0.9	r			Correlation coefficient
	0	x1L			Lower bound on x1
	0.5	x1U			Upper bound on x1
	0	x2L			Lower bound on x2
	1150	x2U			Upper bound on x2
		C	0.76120708		Probability of x1 and x2 within the bounds

The solution is 0.761.

Question 2: Given an upper bound on the weld diameter of 0.5, what upper bound on the shear strength results in a 0.99 probability?

The input of 0.99 is made for C and an iterative solution required for x2U. Using 1150 as the initial guess, the solution is found to be 1156.52.

St	Input	Name	Output	Unit	Comment
		x2U	1156.52373		Upper bound on x2
	0.99	C			Probability of x1 and x2 within the bounds

Question 3: The specification of shear strength calls for a value greater than 1080. If a weld has a diameter of 0.22, what is the probability that the strength requirement will be met?

This is a conditional probability calculation. The sample values are supplied for X1 and X2 and the solution is shown below.

St	Input	Name	Output	Unit	Comment
	0.22	X1			Sample X1 value
		X1L			Lower bound on X1
		X1U			Upper bound on X1
	1080	X2			Sample X2 value
		X2L			Lower bound on X2
		X2U			Upper bound on X2
		Pc1	0.00287229		Probability of X2 given X1
		Pc2	0.67436644		Probability of X1 given X2
		m2c1	1102.91633		Mean of X2 given X1
		s2c1	9.98749218		Standard deviation of X2 given X1
		m1c2	0.08890159		Mean of X1 given X2
		s1c2	0.06164414		Standard deviation of X1 given X2
		Cc1	0.0108808		Probability of X2 or less, given X1

Since we are interested in the probability of a strength *greater* than 1080, we simply take 1-0.01088 and get 0.98912.